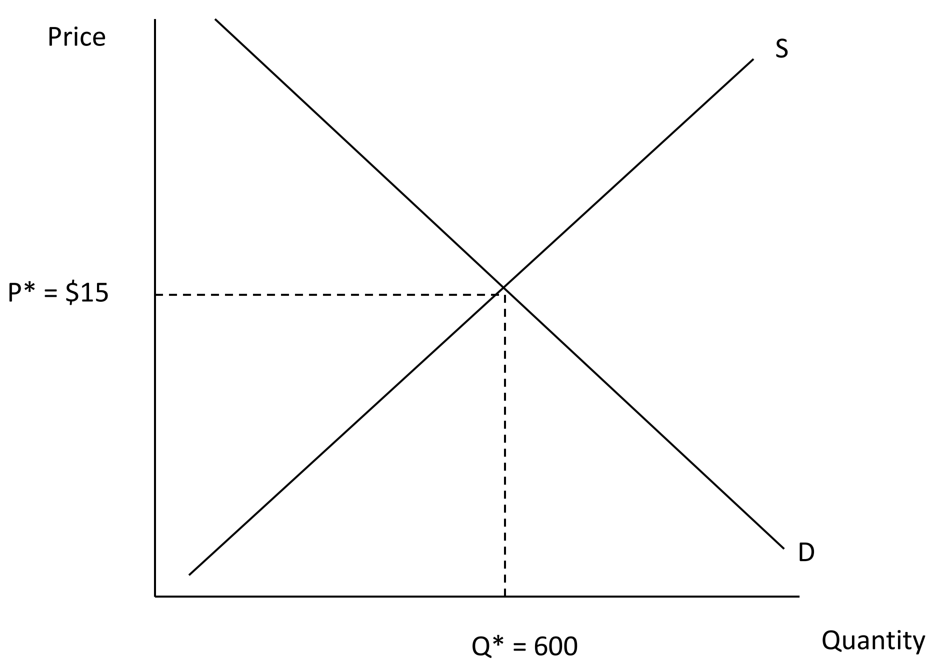
Aggregating Supply and Demand Equations: Private Goods[1]
Instructional Primer[2]

We're often presented with data or graphs that present supply and demand information, but which stop short of providing specified supply or demand equations. Here's a primer to help you better understand the formation of those equations from data you might find in a table or inferred from a graph. Suppose you have the following supply and demand data for the Sandy, Utah market for BBQ Ribs

$Price$	$Quantity_D^{Sandy}$	$Quantity_S^{Sandy}$
$25	200	800
$20	400	700
$15	600	600
$10	800	500
$5	1000	400

From this table we can quickly see that equilibrium in this market is at Price* = $15 and Quantity* = 600 and that it is a strictly linear set of supply and demand curves. We can just as easily graph the data such that we end up with a Supply/Demand Graph representing the Sandy, Utah Market for BBQ Ribs; which looks something like this:



So now let's take this data and form supply and demand equations that might be used more generally. We do this by extending the data table to the point where we see at what price level $Quantity_D^{Sandy} = 0$ and at what

[1] This primer is intended to present an abbreviated discussion of the included economic concepts and is not intended to be a full or complete representation of them or the underlying economic foundations from which they are built.
[2] This primer was developed by Rick Haskell, Ph.D. Student, Department of Economics, College of Social and Behavioral Sciences, The University of Utah, Salt Lake City, Utah (2013)

price level $Quantity_S^{Sandy} = 0$. We note that for each \$5 increase in price we have a decrease in $Quantity_D^{Sandy}$ of 200 units and an increase in $Quantity_S^{Sandy}$ of 100 units; so let's extend the table to the point that each quantity level reaches 0.

$\boldsymbol{Price}$	$\boldsymbol{Quantity_D^{Sandy}}$	$\boldsymbol{Quantity_S^{Sandy}}$
\$30	0	900
\$25	200	800
\$20	400	700
\$15	600	600
\$10	800	500
\$5	1000	400
\$0	1200	300
-\$5	1400	200
-\$10	1600	100
-\$15	1800	0

To form the **Demand Equation** we begin with the form $P = \ (Price\ @\ Q_D^{Sandy} = 0\ ) + \frac{rise}{run}\ Q_D^{Sandy}$. In this case we see that the $\frac{rise}{run}$ for Q_D^{Sandy} is based on the Δ in price over the Δ in Q_D^{Sandy} - which happens to be $-\frac{5}{200} = -\frac{1}{40}$. So where Q_D^{Sandy} is equal to 0, $\boldsymbol{Price = 30 - \frac{1}{40}\ Q_D^{Sandy}}$. This equation is the **Inverse Demand Equation**, or an equation representing demand from the standpoint of price.

To convert this to the **Demand Equation** we simply need to rearrange the equation such that it is in the form of $Q_D^{Sandy} = f(Price, \frac{rise}{run} Q_D^{Sandy})$ as follows:

(1) $Price = 30 - \frac{1}{40}\ Q_D^{Sandy}$

(2) $\frac{1}{40}\ Q_D^{Sandy} = \ 30 - Price$

(3) $Q_D^{Sandy} = \ 1200 - 40\ Price \quad \longrightarrow$ **Demand Equation**

We can set up the **Supply Equation** similarly: we begin with the form $P = \left(Price\ @\ Q_S^{Sandy} = 0\right) + \frac{rise}{run}\ Q_S^{Sandy}$. In this case we see that the $\frac{rise}{run}$ for Q_S^{Sandy} is based on the Δ in price over the Δ in Q_S^{Sandy}- which happens to be $\frac{5}{100} = \ \frac{1}{20}$. So where Q_S^{Sandy} is equal to 0, $Price = -15 + \frac{1}{20}\ Q_S^{Sandy}$ or $\boldsymbol{Price = \frac{1}{20}\ Q_S^{Sandy} - 15}$. This equation is the **Inverse Supply Equation**, or an equation representing Demand from the standpoint of price.

To convert this to the **Supply Equation** we simply need to rearrange the equation such that it is in the form of $Q_S^{Sandy} = f(Price, \frac{rise}{run} Q_S^{Sandy})$ as follows:

(4) $Price = \frac{1}{20}\, Q_S^{Sandy} - 15$

(5) $\frac{1}{20}\, Q_S^{Sandy} = 15 + Price$

(6) $Q_S^{Sandy} = 300 + 20\, Price$ $\longrightarrow$ **Supply Equation**

We can check the accuracy of our equation formation by identifying equilibriating values for *Price* and *Quantity* and comparing them to those we identified through an observation of the table or supply and demand schedule.

(7) $Q_S^{Sandy} = Q_D^{Sandy}$

(8) $300 + 20\, Price = 1200 - 40\, Price$

(9) $60\, Price = 900$

(10) $Price^* = 15$

We can now plug this into either Q_S^{Sandy} or Q_D^{Sandy} to arrive at the equilibrium quantity or Q^*:

(11) $Q_S^{Sandy} = Q^* = 300 + 20\,(15)$

(12) $Q^* = 300 + 300$

(13) $Q^* = 600$

Aggregation of Supply or Demand Equations for private goods

The aggregation of Supply and Demand Equations representative of private goods are bounded by the nature of private goods where $P_{xi} = P_{xj} = P_x$, or the price of good *X* for agent *i* equals the price of good *X* for agent *j* = equals the price of good *X* generally. And $Q_{Xi}^D + Q_{Xj}^D = Q_X^D$, or the quantity demanded of good *X* by agent *i* plus the quantity demanded of good *X* by agent *j* equals the aggregated demand for good *X* generally. **Supply Equations** are treated within the same set of binding constraints as **Demand Equations**. Let's suppose that residents of Draper, Utah also demand some quantity of BBQ Ribs, but they have no independent supply of these Ribs, as represented by the following demand schedule:

$\boldsymbol{Price}$	$\boldsymbol{Quantity_D^{Draper}}$
$25	300
$20	700
$15	1100
$10	1500
$5	1900

With this data we can go about identifying the Demand Equation for BBQ Ribs in Draper in the same manner as we did the demand for BBQ Ribs in Sandy; start by extending the schedule to the point where $Quantity_D^{Draper} = 0$ and then work through the same steps as before:

$\boldsymbol{Price}$	$\boldsymbol{Quantity_D^{Draper}}$
$27 ½	0
$25	100
$20	300
$15	500
$10	700
$5	900

(14) $Price = 27\frac{1}{2} - \frac{5}{200}\, Q_D^{Draper}$

(15) $\frac{1}{40}\, Q_D^{Draper} = 27\frac{1}{2} - Price$

(16) $Q_D^{Draper} = 1100 - 40\, Price \longrightarrow$ **Demand Equation for BBQ Ribs in Draper**

We can then aggregate the Sandy and Draper Demand Equations by summing them as follows:

(17) $Q_D^{Draper} = 1100 - 40\, Price$

(18) + $Q_D^{Sandy} = 1200 - 40\, Price$

(19) $Q_D^{Draper} + Q_D^{Sandy} = 1100 - 40\, Price + 1200 - 40\, Price$

(20) $Q_D = 2300 - Price\,(40 + 40)$

(21) $Q_D = 2300 - 80\, Price$

We can check this by summing the demand schedules for Sandy and Draper as follows:

$\boldsymbol{Price}$	$\boldsymbol{Quantity_D^{Drpaer}}$	$+$	$\boldsymbol{Quantity_D^{Sandy}}$	$=$	$\boldsymbol{Quantity_D}$
$25	100		200		300
$20	300		400		700
$15	500		600		1100
$10	700		800		1500
$5	900		1000		1900

We can then determine the Aggregate Demand Equation in the same manner as before:

$\boldsymbol{Price}$	$\boldsymbol{Quantity_D}$
$28 ¾	0
$25	300
$20	700
$15	1100
$10	1500
$5	1900

(22 $Price = 28\frac{3}{4} - \frac{5}{400}\, Q_D$

(23) $\frac{1}{80}\, Q_D = 28\frac{3}{4} - \frac{1}{80} Price$

(24) $Q_D = 2300 - 80\, Price$ ⟶ **Aggregate Demand Equation for BBQ Ribs**

Which is the same **Demand Equation** we derived in equation (21).

Aggregate Supply and Demand Problem

Suppose the US market for Grits is represented by the follow schedule with quantities presented in milions:

$\boldsymbol{Price}$	$\boldsymbol{Quantity_S^{US}}$	$\boldsymbol{Quantity_D^{US}}$
$2	5	12
$3	10	10

a) Provide completed Supply and Demand Schedules extended to the point at which $\mathrm{Quantity_D^{US}} = 0$ and $\mathrm{Quantity_S^{US}} = 0$.

b) Identify the Inverse Demand and Inverse Supply Equations for this market.

c) What are the Demand and Supply Equations for this market?

d) Based on the Supply and Demand Equations, what are the equilibriating values for price and quantity for this market?

Now suppose that there is also a demand market for Grits in Canada represented by the following schedule:

$\boldsymbol{Price}$	$\boldsymbol{Quantity_D^C}$
$4	12
$5	9

e) What is the Demand Equation for the Canadian market for Grits?

f) Provide an Aggregate Demand Equation for the North American market for Grits by summing the Canadian and US Demand Equations.

g) Show how you would confirm that this equation is accurate by forming an Aggregate Demand Schedule then forming an Aggregate Demand Equation based on that schedule.

h) What are the equalibriating values for price and quantity for the North American Market?

i) What are the quantities demanded by the US and Canadian markets and how do these relate to the aggregate demand quantity?