Annuity Values and Interest Rates[1]
In-Class Problem[2]

In our modern financial markets we often think of an annuity as an insurance product, when in fact it's simply a stream of cash flows (most often constant). When it's for a specified period of time this is simply a form of investment. It's only when one's life expectancy and actual death enter into the equation that it becomes an insurance product.

When we know the expected cash flow, term and rate we can assign a present and future value to the contract, which is simple and obvious enough. What is less obvious is that the rate we consider may not really be the rate experienced by the parties to the contract, due to the potential compounding effects of interest. Compound interest is sometimes referred to as the 8th wonder of the world, when in fact there's little to wonder at if we understand the algebraic concepts underlying the phenomenon.

Suppose you're a consultant advising firms on capital and investment structures. You have a corporate client who's asked you to give an objective analysis of an investment contract being offered with an annual cash flow $6,000 per year, to be paid in equal monthly installments, for a period of 20 years at a stated interest rate of 6%. Your role is to help the client understand what's at play and how to interpret outcomes. With that in mind, consider the following:

a. How much would you recommend paying for this contract? In other words, what is the present value of this time discounted stream of payments? Show both equational and calculator solutions to this problem.

Manual calculation: this is the present value annuity equation

In order to calculate this accurately, we need to recall that 6% per year is 0.5% per month (in simple terms) and that 20 years includes 240 months. So *r* = 0.005 and *t* = 240.

$$PV = PMT\left[\frac{1-\frac{1}{(1+r)^t}}{r}\right] \quad (1)$$

$$= 500\left[\frac{1-\frac{1}{(1.005)^{240}}}{.005}\right]$$

$$= 500\left[\frac{1-\frac{1}{3.3102}}{.005}\right]$$

$$= 69{,}790.35$$

---

[1] This problem and solution set is intended to present an abbreviated discussion of the included finance concepts and is not intended to be a full or complete representation of them or the underlying foundations from which they are built.

[2] This problem set was developed by Richard Haskell, PhD (rhaskell@westminstercollege.edu), Gore School of Business, Westminster College, Salt Lake City, Utah (2015).

Financial Calculator

Make sure P/YR is set to 12 to reflect annual cash flow and that the calculator is set to calculate values as of the end of each period. Then input the following values:

- I/YR = 6
- PMT = 500
- N = 240
- FV = 0

Solve for PV = -69,790.39. This differs from the equational value only by a difference caused by our choice of rounding rules. Notice that this computed value is negative; this is the result of this being an amount we might choose to pay (money out) in exchange for the $500 per month stream of payments (money in).

b. If you were to consider providing the payments under this contract with the intent of building up some future value on which you could might draw, what would be that future value? Show both equational and calculator solutions to this problem.

Manual calculation: this is the future value annuity equation

In order to calculate this accurately, we need to recall that 6% per year is 0.5% per month (in simple terms) and that 20 years includes 240 months. So *r* = 0.005 and *t* = 240.

$$FV = PMT\left[\frac{(1+r)^t-1}{r}\right]$$

$$= 500\left[\frac{(1.005)^{240}-1}{.005}\right]$$

$$= 500\left[\frac{3.3102-1}{.005}\right]$$

= 231,020.00

Financial Calculator

Make sure P/YR is set to 12 to reflect monthly cash flow and that the calculator is set to calculate values as of the end of each period. Then input the following values:

- I/YR = 6
- PMT = -500
- N = 240
- PV = 0

Solve for FV = 231,020.44. This differs from the equational value only by a difference caused by our choice of rounding rules. Notice that this computed value is positive and the payment value is negative; this is the result of this being an amount we would receive (money in) in exchange for the $500 per month stream of payments we would make (money out).

c. **What is the effective annual rate of this contract? This requires manual calculation rather than the use of your financial calculator.**

Since we're dealing with a rate quoted in annual terms but with monthly cash flows, we need to think about the variable *m* representing the number of compounding periods each year: *m* = 12.

$$EAR = \left[1 + \frac{APR}{m}\right]^m - 1$$
$$= \left[1 + \frac{.06}{12}\right]^{12} - 1$$
$$= .06168 \text{ or } 6.17\%$$

d. **Part of your role in to review the details of the investment contract to make sure you and your client are both fully aware of what to expect if this transaction is entered into. While doing so you note that the contract specifies that the 6% rate is an effective rate rather than the annual rate. You mention this to your client, who asks the obvious question, "What's the difference and why do I care?" So, what's the difference and why should your client care?**

The effective interest rate is the rate adjusted for interest's compounding effect. In this case there are 12 compounding periods a year, so we can expect the effective annual rate (EAR) to be higher than what we more commonly think about as the annual percentage rate (APR).

While the difference between 6% and 6.17% is a difference within many people's rounding tolerance, it may not seem like a big deal, but that's because we're only talking about $500 per month. What if we're talking about $5,000,000 per month? In this case the difference is significant in real dollar terms, though seemingly insignificant in relative percentage terms.

e. **If we know the contract has an effective annual rate of 6%, what is the contract's APR?**

Even though we're dealing with a rate quoted in annual terms but with monthly cash flows, we still think about this in terms of annual APR and EAR value. We would expect the APR to be less than the EAR given that we're dealing with monthly compounding. Recall that *m = 12* or the number of compounding periods per year.

$$APR = m\left[(1 + EAR)^{1/m} - 1\right]$$
$$= 12\left[(1.06)^{\frac{1}{12}} - 1\right]$$
$$= .0584 \text{ or } 5.84\%$$