Bond Value and Present Value Calculations[1]
In-Class Problem[2]

Suppose you're the CFO for a firm seeking to raise $25,000,000 in debt capital. You've chosen to go to the bond market for this round of funding and are working with an investment firm to represent your bonds in the open market. The firm also acts in an advisory capacity with a fiduciary responsibility to help you structure your offering. Your firm has been given an AA credit rating from Standard and Poor's and you've been advised to issue 20-year bonds with semi-annual interest payments, a 6% coupon rate, and a face value of $1,000 per bond. You now go through the process of filing your bond debenture with the SEC and state securities regulators and await the date of your bond offering.

a. **Assuming your bonds are issued as planned, what is the effective annual rate (EAR) investors will receive and why might it be different than the coupon rate?**

$$EAR = \left[1 + \frac{APR}{m}\right]^m - 1 \; = \; \left[1 + \frac{.06}{2}\right]^2 - 1$$

$$= .0609 \text{ or } 6.09\%$$

b. **If your bonds are issued at their face value, what does it tell you about the expected yield in the market at the time, and what is that YTM?**

It tells us that the expected market yield (YTM) is the same as the coupon rate on the bonds. It is 6.00%.

c. **Suppose your bonds are issued at $994 rather than $1,000 each, what does this tell you about the expected market yield at the time. Be specific here, this requires both a conceptual and numeric response.**

We would expect this to inform us that market yield expectations are greater than 6% (as market rates rise, bond values fall). We can calculate this with a simple YTM calculation on our HP10bII calculators using the following variable structure:

P/YR	2	Solve for I/YR = 6.0521%
N	40	
PMT	30	
PV	-994	
FV	1000	

d. Suppose you're now considering the value of your firm's bonds 5- years after their date of issue in a market where yield expectations are 3%. What would you expect to be the value of each bond? Calculate this using the relevant equation and then use your calculator to confirm the accuracy of your response.

$$BV = C\frac{\left[1-\frac{1}{(1+YTM)^t}\right]}{YTM} + \frac{F}{(1+YTM)^t}$$

$$BV = 30\frac{\left[1-\frac{1}{(1.015)^{30}}\right]}{.015} + \frac{F}{(1.015)^{30}} = 30\frac{\left[1-\frac{1}{1.563}\right]}{.015} + \frac{1000}{1.563}$$

BV = 720.409 + 639.795 = $1,360.204

e. If an investor were to purchase one of your bonds at issue for $994, hold it for 12 years and then sell in the open market having received a return of 8%, for how much would the bond need to be sold?

$$BV = C\frac{\left[1-\frac{1}{(1+YTM)^t}\right]}{YTM} + \frac{F}{(1+YTM)^t}$$

$$994 = 30\frac{\left[1-\frac{1}{(1.04)^{24}}\right]}{.04} + \frac{F}{(1.04)^{24}} = 30\frac{\left[1-\frac{1}{2.5633}\right]}{.04} + \frac{F}{2.5633} = 457.40 + \frac{F}{2.5633}$$

$$994 - 457.40 = 536.60 = \frac{F}{2.5633}$$

F = 2.5633 x 536.60 = 1,375.446

So the bond would need to be sold for a premium of 375.47 for you to receive 8% during your holding period. But I thought that as rates increase after a bond is issued the value of the bond decreases; so how can this be? You've got to think about this differently than might be immediately obvious. This scenario has to do with what overall rate did the buyer receive rather than what YTM might be active in a market at a point in time.

f. Based on the information provided in (e) above, what would have to be the market YTM for a bond to have a value of $1,375.446?

We now have to think from the perspective of the scenario at the time this bond might have this value of $1,375.446. If the 20 yr. bond had been held for 12 years it then has 8 years remaining, so N = 8 x 2 = 16. From there we can simply use the calculator and solve for I/YR which will also be the current market YTM as follows.

P/YR	2	Solve for I/YR = 1.087%
N	16	
PMT	30	

PV   -1375.446
FV   1000

Rates would need to fall from their level of 6.0521% at issue date to 1.087% just twelve years later. Is this realistic, sure it is. We just went through a similar rate change in the market thanks to the financial crisis and the way the Federal Reserve has sought to stimulate the markets.

g. **If a bond a bond paying twice a year has an effective annual rate of 1.0899%, what would be the bond's APR?**

$$APR = m\left[(1 + EAR)^{\frac{1}{m}}\right] - 1$$

$$APR = 2\left[(1 + .010899)^{\frac{1}{2}} - 1\right]$$

$$APR = 2[1.005435 - 1] = .0108695 \text{ or } 1.087\%$$

h. **What would need to be the APR of a municipal bond issued in the state of a tax payer's residence need to pay in order to provide the same yield of a corporate bond paying 6.0521%, assuming the tax payer has a tax rate of 24%**

The equation[3] for this is $r_{MUNI} = (r_{CORP})(1 - r_{TAX})$

$$r_{MUNI} = (0.060521)(1 - 0.24)$$

$$= 0.060521 \times .76 = 0.04599 \text{ or } 4.599\%$$

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[3] $r_{MUNI} = (r_{CORP})(1 - r_{TAX})$ can be derived as follows:
  After tax municipal bond coupon payment = $(F)(r_{MUNI})(1 - 0) = (F)(r_{MUNI})$
  After tax corporate bond coupon payment = $(F)(r_{CORP})(1 - r_{TAX})$
  We need to set these equal to each other
  $(F)(r_{MUNI}) = (F)(r_{CORP})(1 - r_{TAX})$
  Divide both sides by F to arrive at $r_{MUNI} = (r_{CORP})(1 - r_{TAX})$