Breakeven Analysis[1]
In-Class Problem[2]

The subject firm for the problems represented in this case is Almaden Technologies, a fictional firm for which hypothetical values have been presented. Financial and production data presented are used solely in support of building a body of business fundamentals problem sets and case studies.

Suppose that you're an Operations Manager for Almaden Technologies (AT) and you've been tasked to identify the breakeven point on the production of a new internet connected home automation device, "Lights", currently under consideration and priced at $190.00 to the firm's network of distributors, who sell it to the public based on MSRP of $350.00. The firm has allocated a production/warehousing facility with a 100,000 unit monthly output capacity based on running an 8 hour shift each workday and has estimated the following costs with respect to the project:

Monthly Costs		Per Unit Costs: "Lights"	
Building	**250,000**	**Production labor**	**45.00 per unit**
Project Management	**175,000**	**Production supervisors**	**5.00 per unit**
Advertising	**300,000**	**Shipping**	**8.00 per unit**
Computer network	**75,000**	**Sales Commissions**	**12.00 per unit**
General Administrative	**150,000**	**Materials**	**70.00 per unit**
Equipment (depreciation)	**200,000**	**Energy costs**	**3.00 per unit**
		Wholesale price	**190.00 per unit**

1. What is the value of TFC for this project?

TFC = Bldg + Project Mgt + Advertising + Computer Network + General Admin + Equipment (Dep)

= 250,000 + 175,000 + 300,000 + 75,000 + 150,000 + 200,000

= 1,150,000

2. What is the value of AVC for this project?

Since the variable costs indicated are listed in per unit values, AVC can be stated as follows:

AVC = Prod Labor + Prod Supv + Shipping + Sales Comm + Materials + Energy Cost

= 45.00 + 5.00 + 8.00 + 12.00 + 70.00 + 3.00

= 143.00

[1] This problem and solution set is intended to present an abbreviated discussion of the included finance concepts and is not intended to be a full or complete representation of them or the underlying foundations from which they are built.

[2] This problem set was developed by Richard Haskell, PhD (rhaskell@westminstercollege.edu), Gore School of Business, Westminster College, Salt Lake City, Utah (2016).

3. Based on the information provided, what quantity must this firm produce in order to be at breakeven?

The formula for Breakeven is as follows:

$$0 = (\mathrm{P})(\mathrm{Q}) - \mathrm{TFC} - (\mathrm{AVC})(\mathrm{Q}) \quad (1)$$

This can be rearranged as

$$0 = (\mathrm{P} - \mathrm{AVC})(\mathrm{Q}) - \mathrm{TFC}$$

Normalize on Q (rearrange to solve for Q) as follows

$$(\mathrm{P} - \mathrm{AVC})(\mathrm{Q}) = \mathrm{TFC}$$

$$\mathrm{Q} = \frac{\mathrm{TFC}}{\mathrm{P}-\mathrm{AVC}} \quad (2)$$

Solve for Q by substituting known values

$\mathrm{Q} = \frac{1{,}550{,}000}{190-143}$ = 24,468.09

Since there's little benefit in producing 0.09 units, we round this up to the nearest whole unit to arrive at

Q = 24,469

4. If the firm produces at the plant's capacity, what is the minimum price at which the firm can sell the product and still breakeven?

In this case we'll substitute 100,000 for Q into (2) and rearrange the equation to solve for P

$$\mathrm{Q} = \frac{\mathrm{TFC}}{\mathrm{P}-\mathrm{AVC}}$$

$$(\mathrm{P} - \mathrm{AVC})(\mathrm{Q}) = \mathrm{TFC}$$

$$\mathrm{P} - \mathrm{AVC} = \frac{\mathrm{TFC}}{\mathrm{Q}}$$

$$\mathrm{P} = \frac{\mathrm{TFC}}{\mathrm{Q}} + \mathrm{AVC}$$

$\mathrm{P} = \frac{1{,}150{,}000}{100{,}000} + 143$ = 154.50

5. **If the firm produces and sells as planned at the plant's current capacity, how much profit would the firm be expected to produce with this project?**

Substitute known values in (1) to solve for Profit (π)

$\pi = (\mathrm{P})(\mathrm{Q}) - \mathrm{TFC} - (\mathrm{AVC})(\mathrm{Q})$ (3)

$\pi = (190.00 - 143.00)(100{,}000) - 1{,}150{,}000 = 3{,}550{,}000$

$\pi = 3{,}550{,}000$

6. **Suppose the firm seeks to target profit of $3,000,000 from this product based on the input costs and wholesale price noted in the problem narrative. How many units would the firm need to produce to generate the required profit?**

Substitute known values into (3) and solve for Q

$\pi = (\mathrm{P})(\mathrm{Q}) - \mathrm{TFC} - (\mathrm{AVC})(\mathrm{Q})$

$3{,}000{,}000 = (190)(\mathrm{Q}) - 1{,}150{,}000 - (143.00)(\mathrm{Q})$

$3{,}000{,}000 = (190 - 143.00)(\mathrm{Q}) - 1{,}150{,}000$

$3{,}000{,}000 + 1{,}150{,}000 = (190.00 - 143.00)(\mathrm{Q})$

$\frac{4{,}150{,}000}{190.00-143.00} = \mathrm{Q} = 88{,}297.97$

Again, since this fractional unit isn't of any real value, we round to the next whole unit

$\mathrm{Q} = 88{,}298$

As a result of the successful launch and sale of "Lights", the firm's management has decided to begin production of a second internet connected home automation device, "Sounds", which the marketing group is confident will sell alongside "Lights" at a price point of $200. Marketing has prepared a demand analysis for "Sounds" and determined that for each unit of "Lights" sold one unit of "Sounds" will also sell such that $Q_{Lights} = Q_{Sounds}$. In order to accommodate the new product the production/warehouse facility will be redesigned to run two shifts each day with each shift committing 50% of its production to "Lights" and 50% to "Sounds".

You've determined the facility redesign will cost $1,800,000 and have been informed by accounting that this cost will be amortized (accounted for) over a period of 36 months. In addition, monthly equipment expense (depreciation) will rise to $400,000, advertising will need to increase by $200,000, and Project Management and General Administrative will each increase by $75,000. Finally, you've been given the following variable costs for "Sounds":

Per Unit Costs: "Sounds"	
Production labor	**55.00 per unit**
Production supervisors	**5.00 per unit**
Shipping	**8.00 per unit**
Sales Commissions	**10.00 per unit**
Materials	**80.00 per unit**
Energy costs	**3.00 per unit**

7. How many units of "Lights" and "Sounds", each, must the firm now produce to at breakeven?

First we now have an additional unknown for which we'll need to solve – in this case we have Q_{Lights} and Q_{Sounds}, but we've also been told that $Q_{Lights} = Q_{Sounds}$ so we also have an additional equation to help us with the solution.

Let's consider values for TFC and AVC_{Sounds} as follows:

TFC = Bldg + Project Mgt + Advertising + Comp Network + General Admin + Equip (Dep) + Redesign

= 250,000 + 250,000 + 500,000 + 75,000 + 225,000 + 400,000 + 50,000

= 1,750,000

AVC_{Sounds} = Prod Labor + Prod Supv + Shipping + Sales Comm + Materials + Energy Cost

= 55.00 + 5.00 + 8.00 + 10.00 + 80.00 + 3.00

= 161.00

Now let's rethink the equation starting with (1)

$0 = (\mathrm{P})(\mathrm{Q}) - \mathrm{TFC} - (\mathrm{AVC})(\mathrm{Q})$

Expand to include new variables

$$0 = (\mathrm{P}_{Lights})(\mathrm{Q}_{Lights}) + (\mathrm{P}_{Sounds})(\mathrm{Q}_{Sounds}) - \mathrm{TFC} - [(\mathrm{AVC}_{Lights})(\mathrm{Q}_{Lights}) + (\mathrm{AVC}_{Sounds})(\mathrm{Q}_{Sounds})]$$

Substitute $\mathrm{Q}_{\mathrm{Lights}}$ for $\mathrm{Q}_{\mathrm{Sounds}}$ as indicated by the new equation

$$0 = (\mathrm{P}_{Lights})(\mathrm{Q}_{Lights}) + (\mathrm{P}_{Sounds})(\mathrm{Q}_{Lights}) - \mathrm{TFC} - [(\mathrm{AVC}_{Lights})(\mathrm{Q}_{Lights}) + (\mathrm{AVC}_{Sounds})(\mathrm{Q}_{Lights})]$$

Simplify the equation

$$0 = (\mathrm{P}_{Lights} + \mathrm{P}_{Sounds} - \mathrm{AVC}_{Lights} - \mathrm{AVC}_{Sounds})(\mathrm{Q}_{Lights}) - \mathrm{TFC} \quad (4)$$

Substitute known values and normalize on $\mathrm{Q}_{\mathrm{Lights}}$

$$0 = (190.00 + 200.00 - 143.00 - 161.00)(\mathrm{Q}_{Lights}) - 1{,}750{,}000$$

$$1{,}750{,}0000 = (86.00)(\mathrm{Q}_{Lights})$$

$$\frac{1{,}750{,}000}{86.00} = \mathrm{Q}_{Lights} = 20{,}348.84$$

Recall that $\mathrm{Q}_{\mathrm{Lights}} = \mathrm{Q}_{\mathrm{Sounds}}$ such that $\mathrm{Q}_{Sounds} = 20{,}348.84$ also

We should think of these values as $\mathrm{Q}_{\mathrm{Lights}} = \mathrm{Q}_{\mathrm{Sounds}} = 20{,}349$ since a fractional unit is of no real value to us

8. If the firm targets a profit of $5,000,000 from the entire operation how many units of "Lights" and "Sounds", each, must be produced?

Substitute known values into (4) and solve for quantities

$$0 = (\mathrm{P}_{Lights} + \mathrm{P}_{Sounds} - \mathrm{AVC}_{Lights} - \mathrm{AVC}_{Sounds})(\mathrm{Q}_{Lights}) - \mathrm{TFC}$$

$$5{,}000{,}000 + 1{,}750{,}000 = (190.00 + 200.00 - 143.00 - 161.00)(\mathrm{Q}_{Lights})$$

$$6{,}750{,}000 = (86.00)(\mathrm{Q}_{Lights})$$

$$\frac{6{,}750{,}000}{86.00} = \mathrm{Q}_{Lights} = 78{,}488.37$$

Once again, recall that $\mathrm{Q}_{\mathrm{Lights}} = \mathrm{Q}_{\mathrm{Sounds}}$ such that $\mathrm{Q}_{Sounds} = 78{,}488.37$

We should think of these values as $\mathrm{Q}_{\mathrm{Lights}} = \mathrm{Q}_{\mathrm{Sounds}} = 78{,}489$ since a fractional unit is of no real value to us