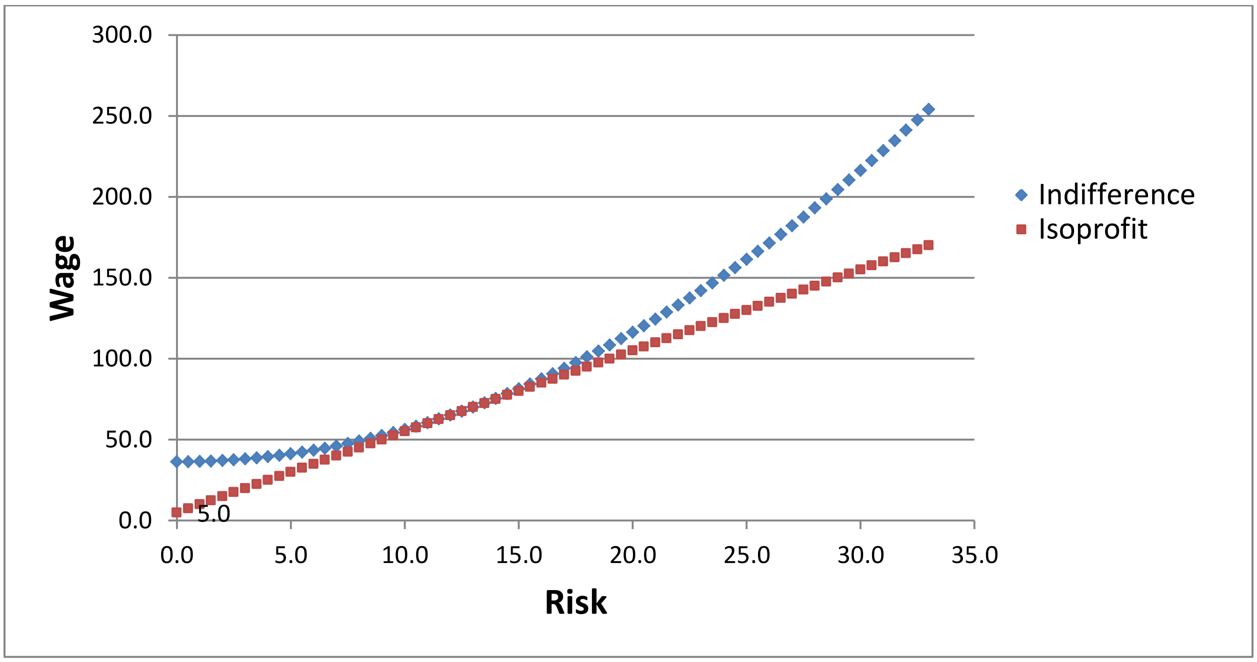
Compensating Wage Differential[1]
In-Class Problem[2]

Let's assume that we have a worker with an indifference curve parameterized by the equation $W = 36.25 + .2R^2$ and that we have a firm with an Isoprofit Curve parameterized by the equation $R = \frac{1}{5}W - 1$; where W = wage and R = Risk. In this problem, the firm is in a competitive goods and labor market.

How would we think about this sort of problem?

Let's first think of the type of graphic model this might represent. The only two variables we have are W and R (Wage and Risk) and we're familiar with a model with Wage on the Y axis and Risk on the X axis; a particular form of a Compensating Wage Differential model. We also know that a worker indifference curve for this type of model will be convex and upward sloping such that shifts up and to the left represent higher levels of worker utility. We further know that the firm's Isoprofit curve will also be upward sloping and is either concave or quasi-concave, and that at any point on the curve the firm has the exact same level of profit. Finally we know that if the firm and worker "match", then there will be a point of tangency between the two curves.



With this model in mind we can ask a few questions:

1. Will the worker choose to work for this firm?
2. If so what is the optimal combination of risk and reward the worker will accept and what is the optimal level of risk and profit the firm will accept?

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[1] This problem is intended to present an abbreviated discussion of the included economic concepts and is not intended to be a full or complete representation of them or the underlying economic foundations from which they are built.

[2] This In-Class Problem was developed by Rick Haskell (rick.haskell@utah.edu), Ph.D. Student, Department of Economics, College of Social and Behavioral Sciences, The University of Utah, Salt Lake City, Utah (2014).

We should first see if the worker will even choose to work for this firm by setting the two equations equal to each other. To do this we need to have each equation normalized on the same variable and it looks like it would be easiest to normalize both on W. So let's start by rearranging the Isoprofit equation:

$R = \frac{1}{5}W - 1$ can be rearranged as $W = 5 + 5R$

Since W = W we can now set the two equations equal to each other such that Isoprofit = Indifference:

$36.25 + .2R^2 = 5 + 5R$

This isn't going to reveal a simple solution for R that we can then plug back into one of the two equations. In fact, it forms a quadratic equation that needs to be solved via the quadratic formula

We can rewrite the equation in quadratic form as $aR^2 + bR + c = 0$

$31.25 - 5R + .2R^2 = 0$ which can be rearranged as $.2R^2 - 5R + 31.25 = 0$, such that a = .2, b = -5 and c = 31.25.

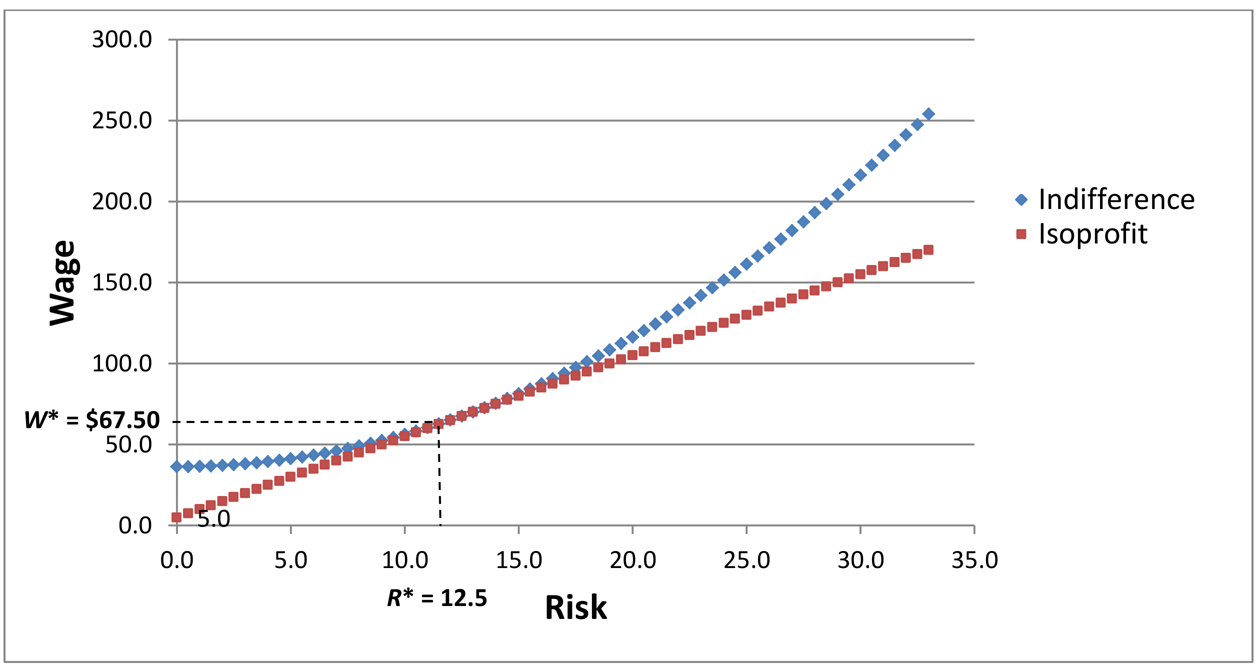
We can solve this through the quadratic equation, $R = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ as follows:

$R = \frac{5 \pm \sqrt{5^2-4(.2)(31.25)}}{2(.2)} = \frac{5 \pm \sqrt{25-25)}}{.4} = \frac{5}{.4}$ = 12.5 = R^*.

We can the plug this value into either of the originating equations; for simplicity we'll use the Isoprofit normalized on W:

$W = 5 + 5R = 5 + 5(12.5) = 67.5 = W^*$.

So we have a tangency of these two curves at $R^* = 12.5$ and $W^* = \$67.50$ which tells us that we can match this firm and this worker with the worker accepting a risk level of 12.5 and a wage of \$67.50.



So what does this tell us about the firm's profit level? Since we're in a competitive market, it tells us that the profit level = 0, so the firm is prepared to mitigate risk to a level of 12.5 and have a profit level of 0.

What if the we can't obtain a point of tangency, if we can't find an equilibriating value for R and W? It simply means that this firm and worker can't come to an agreement and there will be labor market transaction between them.