

Consumer Budgets, Indifference Curves, and Utility Maximization¹

In-Class Problem²

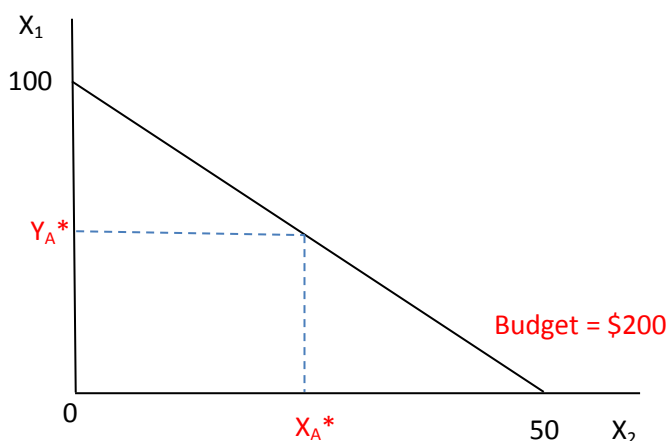
Let's assume we have a consumer with a budget of \$200 and for whom we have a marginal rate of substitution of $X_2/2X_1$ where the price of good $X_1 = \$2$ and the price of good $X_2 = \$4$. Also suppose that this consumer is a rational, self-interested and utility maximizing economic agent.

- a. Provide a complete consumer model showing the consumer's budget and potential choices of X_1 and X_2 , be as specific as possible, but only represent the information we know. For consistency, place X_1 on the Y axis and X_2 on the X axis.

To identify the values of X_1 and X_2 on their respective axes, divide the budget by the price of the good:

$$X_1 = \frac{\$200}{\$2} = 100$$

$$X_2 = \frac{\$200}{\$4} = 50$$



- b. What would be this consumer's optimal consumption of goods X_1 and X_2 ? Both specify these levels and show them in detail on the model in part a. Be sure to include some graphic representation of the consumer's tastes and preferences in this model.

$$P_{X_1}(X_1) + P_{X_2}(X_2) = M = \$200 \quad \text{This is simply the consumer's budget} \quad (1)$$

$$P_{X_1} = \$2 \quad \text{and} \quad P_{X_2} = \$4 \quad \text{These are the prices for two goods} \quad (2)$$

$$MRS = \frac{P_{X_1}}{P_{X_2}} = \frac{X_2}{2X_1} \quad \text{The MRS was stated above and we simply equate} \quad (3)$$

it to the MRS through a relationship between the two goods informing us of a particular form of consumer utility

¹ This primer is intended to present an abbreviated discussion of the included economic concepts and is not intended to be a full or complete representation of them or the underlying economic foundations from which they are built.

² This In-Class Problem was developed by Rick Haskell (rick.haskell@utah.edu), Ph.D. Student, Department of Economics, College of Social and Behavioral Sciences, The University of Utah, Salt Lake City, Utah (2014).

This is a direct result of the balance condition for utility maximization, which yields the marginal rate of substitution (MRS).

$$\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y} \quad (4)$$

Which can be rewritten as

$$\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y} \quad (5)$$

Since this is a utility maximizing condition, it's a target we want to achieve and as such governs much of the rest of what we'll do: remember that it simply tells us something about the marginal utilities of X and Y in respect to their prices. We have another equation (3) that also tells us something about X and Y in respect to their prices so let's rewrite these. Since we know the prices of these inputs we'll first substitute (2) into (3)

$$\frac{2}{4} = \frac{1}{2} = \frac{X_2}{2X_1} \quad (6)$$

Which can be rewritten as

$$X_2 = \left(\frac{1}{2}\right)(2X_1) = X_1 \quad (7)$$

If we replace P_{X1} and P_{X2} with the values given in (2) we can rewrite the equation as

$$2X_1 + 4X_2 = 200 \quad (8)$$

Now we have two equations (7) and (8) and two unknowns (X_1 and X_2) from which we're going to find the optimal values of X_1 and X_2 this consumer will use to achieve maximum utility given a budget constraint.

We can substitute (7) into (8) such that

$$2X_1 + 4X_1 = 200 \quad \text{or} \quad 6X_1 = 200$$

From here we can rewrite the equation as

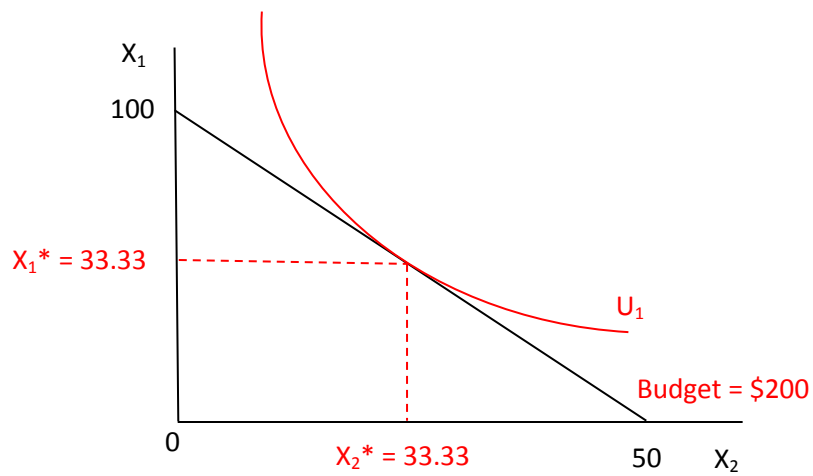
$$X_1 = \frac{200}{6} = 33.33 \quad \text{or} \quad X_1^* = 33.33 \quad (9)$$

Now we simply need to substitute (9) into (8) to get

$$2(33.33) + 4X_2 = 200$$

$$4X_2 = 133.33$$

$$X_2^* = 33.33$$



- c. Suppose the price of X_1 changes to \$3; how would this effect the consumer's consumption of X_1 and X_2 . Think about this in general terms without actually calculating X_1^* and X_2^* , but show the changes in how many units of X_1 and X_2 that might be purchased and then show any generally expected or potential movement in the consumer's utility.

