**Graphing Point-Slope and Quadratic Equations**[1]
**In-Class Problem**[2]

**Section I:** Suppose you've been provided with the following inverted demand and supply curves for a product in a particular market: $P = 16 - \frac{1}{4} Q_D$ and $P = 9\frac{1}{3} + \frac{1}{6} Q_S$. As the head of the COO of a firm in this industry you're responsible to identify the number of units the firm will produce in order for the firm to sell the highest possible quantity in the consumer market. You understand this will require that you price the product as low as possible without losing money to motivate sales. As such the price of the product to consumers will equal the cost of the product to produce. You're responsible to graph this relationship for other firm management as normally constructed supply and demand curves.

1. **Specify the normally constructed demand and supply equations.**

$P = 16 - \frac{1}{4} Q_D$ *the firm's inverted demand equation* (1)
*Rearrange equation by subtracting P and adding ¼ $Q_D$ from each side of the equation*
$\frac{1}{4} Q_D = 16 - P$
*Multiply both sides of the equation by 4*
**$Q_D = 64 - 4P$** ***this is the firm's demand equation***

$P = 9\frac{1}{3} + \frac{1}{6} Q_S$ *the firm's inverted supply equation* (2)
*Rearrange equation by subtracting $9\frac{1}{3}$ from each side of the equation*
$\frac{1}{6} Q_S = -9\frac{1}{3} + P$
*Multiply both sides of the equation by 6*
**$Q_S = -56 + 6P$** ***this is the firm's supply equation***

2. **Identify the point on the graph at which $Q_S = Q_D$. The resultant values of Q and P are referred to as Q* and P* and are the equilibrium or market clearing values of Q and P.**

Start by setting the equations equal to one another and rearrange to solve for P as follows:

$Q_S = Q_D$
$-56 + 6P = 64 - 4P$
$10P = 120$
$P^* = \frac{120}{10} = 12$

[1] This problem and solution set is intended to present an abbreviated discussion of the included finance concepts and is not intended to be a full or complete representation of them or the underlying foundations from which they are built.
[2] This problem set was developed by Richard Haskell, PhD (rhaskell@westminstercollege.edu), Gore School of Business, Westminster College, Salt Lake City, Utah (2016).

The value of $P^*$ is the P at which $Q_S$ and $Q_D$ are equal, so we can substitute $P^* = 12$ into either the $Q_S$ or $Q_D$ equation to solve for $Q^*$:

$Q^* = 64 - 4(12) = 16$

**3. Identify the X and Y axis intercepts of the supply and demand equations.**

Do this by setting P = 0 and then Q = 0 for each equation as follows:

| $\underline{Q_S = -56 + 6P}$ | | $\underline{Q_D = 64 - 4P}$ | |
|---|---|---|---|
| If P = 0 | $Q_S = -56 + 6(0)$ | If P = O | $Q_D = 64 - 4(0)$ |
| | $Q_S = -56$ | | $Q_D = 64$ |
| If $Q_S = 0$ | $0 = -56 + 6P$ | If $Q_D = 0$ | $0 = 64 - 4P$ |
| | $6P = 56$ | | $4P = 64$ |
| | $P = 9\frac{1}{3}$ | | $P = 16$ |

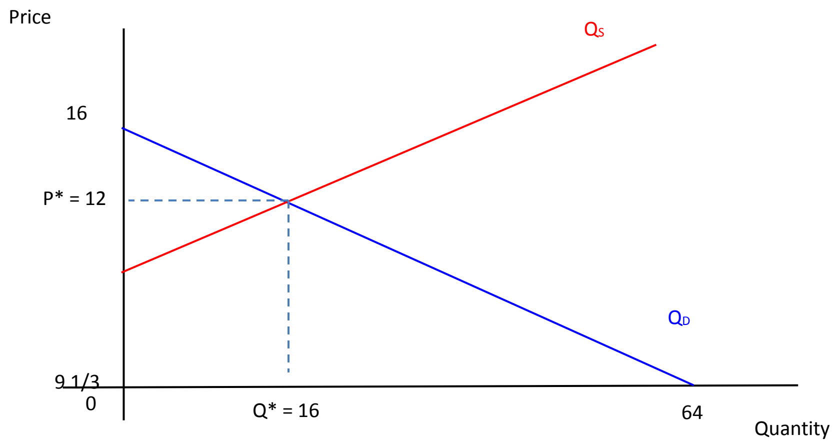
**4. Construct a table of values for each of the equations identified in #1 starting with P = \$9 and increasing P by \$1 until P = \$16. At what price is $Q_S = Q_D$**

| | | Constant | Scalar |
|---|---|---|---|
| **Demand** | $Q_D = 64 - 4P$ | 64 | -4 |
| **Supply** | $Q_S = -56 + 6P$ | -56 | 6 |

| **Demand** | | **Supply** | |
|---|---|---|---|
| **P** | **$Q_S$** | **P** | **$Q_D$** |
| 9.00 | 28.00 | 9.00 | -2 |
| 10.00 | 24.00 | 10.00 | 4 |
| 11.00 | 20.00 | 11.00 | 10 |
| 12.00 | 16.00 | 12.00 | 16 |
| 13.00 | 12.00 | 13.00 | 22 |
| 14.00 | 8.00 | 14.00 | 28 |
| 15.00 | 4.00 | 15.00 | 34 |
| 16.00 | 0.00 | 16.00 | 40 |

$Q_S = Q_D$ at $P^* = 12$, $Q^* = 16$

5. **Provide a graphic model of the equations in positive space. Be sure to completely label all relevant items completely.**




*Notice that we could have graphed the curves for $Q_S$ and $Q_D$ by using the table of values and plotting points to arrive at the formed curves, or we could have simply used the axes intercepts and equilibrium values to arrive with the exact same graphic model. This is the result of $Q_S$ and $Q_D$ being linear equations in this case, each forming straight lines.*

**Section II:** Suppose you're the senior production manager of a manufacturing firm in which the workers belong to a union for which the contract is currently being renegotiated with all firms in your part of the country. Your firm's senior management would like to avoid a strike and has come to you with the union's most recent offer with the request to analyze it and report back. You know the metrics of your firm result in a labor demand equation parameterized by the equation $L_D = 500 - 10W + .05W^2$ and the union's offer is contained the equation $L_S = -50 + 5W + .05W^2$. You've chosen to identify the number of workers your firm should employ and at what wage they'll be paid in order to determine whether or not the offer is acceptable to the firm.

1. **Identify the equilibrium values for Labor (L) and Wage (W).**

   LS = LD
   $500 - 10W + .05W^2 = -50 + 5W + .05W^2$
   Add $.05W^2$ to each side, *effectively cancelling out the quadratic in this solution*
   Add 10W and 50 to each side to arrive at
   15W = 550 divide both sides by 15
   W* = 36.67

   To solve L* let's use each of the equation
   $L^* = LD = -50 + 5(36.67) + .05(36.67^2) = 200.55$
   $L^* = LS = 500 – 10(36.67) + .05(36.67^2) = 200.55$

2. **Form a table of values for W, LS, and LD starting with W = $0 through W = $100. You can do this in increments of $10 each.**

| W | LS | LD |
|---|---|---|
| 0 | -50 | 500 |
| 10 | 5 | 405 |
| 20 | 70 | 320 |
| 30 | 145 | 245 |
| 40 | 230 | 180 |
| 50 | 325 | 125 |
| 60 | 430 | 80 |
| 70 | 545 | 45 |
| 80 | 670 | 20 |
| 90 | 805 | 5 |
| 100 | 950 | 0 |

3. **Provide a graphic model based on the LS and LD equations. In this case you'll want to plot the values used in the table, but be sure to accurately label all relevant values (axes intercepts, equilibrium levels, curve labels, etc). This is why we perform the equilibrium analysis first.**