**Solving Basic Equations – Solution Set[1]**
**In-Class Problem[2]**

Provide answers to each of the following questions based on the information provided. When forming a graph or model it is not necessary to provide perfectly proportioned graphs. It is necessary to accurately present all relevant labels and to include all available or calculable data points and their respective values.

1. **Consider a model in which the demand for high quality golf shirts is represented through the equation $Q_D = 250 - 12P$ in which P = price of golf shirts and Q = quantity of golf shirts in dozens. The supply of golf shirts is represented by the equation QS = -150 + 8P.**

    a. **What is the equilibrium value for the price golf shirts (P*)?**

    $Q_S = Q_D$
    -150 + 8P = 250 – 12P
    20P = 400
    P* = 20

    b. **What is the market clearing quantity of golf shirts (Q*)?**

    $Q_D = 250 - 12(20) = 10$
    $Q_S = -150 + 8(20) = 10$
    Q* = 10

2. **Suppose you know that the total revenue (TR) of a firm less its total costs (TC) results in a $1,200 profit on sales (TR – TC = $1,200). The firm produces and sells water bottles at $6.00, has fixed costs of $100 and variable costs of $2.75 per bottle. How many water bottles will the firm need to sell to meet its profit target?**

    TR = P * Q = 6Q
    TC = Fixed Costs + (Variable Costs x Q) = 100 + 2.75 Q
    TR – TC = $1,200
    6Q – (100 + 2.75Q) = 1200
    6Q – 2.75Q -100 = 1200
    3.25Q = 1300
    Q = 400

[1] This problem and solution set is intended to present an abbreviated discussion of the included finance concepts and is not intended to be a full or complete representation of them or the underlying foundations from which they are built.
[2] This problem set was developed by Richard Haskell, PhD (rhaskell@westminstercollege.edu), Gore School of Business, Westminster College, Salt Lake City, Utah (2016).

3. **Suppose you've been given a wage and labor relation for electronics assembly workers in a particular labor market parameterized by the equations $L_D = 86 + -4W^2$ and $L_S = 5 + 5W$ where W is in hundreds of dollars per week and L is workers in 100's.[3] What would be the market clearing wage and quantity of labor for this market?**
**In order to solve this we need to set the two equations equal to one another and solve for W. In this case this results in a quadratic equation requiring the quadratic formula to solve as follows:**

$L_S = L_D$
$5 + 5W = 86 + -4W^2$
In this case we can't simply isolate W, but need to recognize that the presence of both the variable (W) and the variable squared (W2) sets up a quadratic equation in the following form
$0 = -4W^2 - 5W + 81$

Recall that a quadratic equation can in the form of $\mathbf{0 = aX^2 + bX + c}$ and that it can be solved by using the quadratic formula $\boldsymbol{X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$.

In this construction a = -4, b = -5 and c = 81. Substitute these values into the quadratic equation to solve for the possible values of X.

$$\boldsymbol{W = \frac{-(-5) \pm \sqrt{-5^2 - 4(-4)(81)}}{2(-4)}}$$

$$\boldsymbol{W = \frac{5 \pm \sqrt{25 + 1296}}{-8} = \frac{5 \pm \sqrt{1321}}{-8}} = \frac{5 \pm 36.345}{-8}$$, which has two possible solutions as a result of the ± operation.

$$\boldsymbol{W = \frac{5 + 36.354}{-8} = -5.1682} \quad \text{and} \quad \boldsymbol{W = \frac{5 - 36.354}{-8}} = 3.9182$$

So we see that we've calculated a positive and a negative result. Before seeking to solve for L* we have to determine which W* to use, which in this case is somewhat obvious as W* = -5.1682 isn't plausible and W* = 3.9182 is. Since the positive iteration results in a negative value for W*, we consider this value an irrelevant alternative and discard it.

Now substitute the positive value for W* into either the $L_S$ or $L_D$ equation and solve for L*

$$\boldsymbol{L_S = 5 + 5(3.9182) = 24.59 = L^*}$$

$$\boldsymbol{L_D = 86 - 4(3.9182)^2 = 24.59 = L^*}$$

[3] **The somewhat more complex $L_D$ relation could be a function of a more complex cost structure for firms in the market**