The Producer's Problem – Optimal Levels of K and L[1]

Instructional Primer[2]

The producer is faced with many decisions of course, but among the most important is how to keep production costs at a minimum while keeping output at a maximum. This is consistent with profit maximization in a competitive market environment of course, but also represents choosing the least cost technology to obtain the desired level of output given available factor inputs at prices the producer has little or no influence over (assuming perfect market conditions). This leads to using optimal levels of those factor inputs, which are often simply thought of as capital (K) and labor (L).

In this example we'll use a few pieces of information that must be given and then apply them to a set of rules or conditions that we've learned or developed. In this case the given information is simply arbitrarily selected, but in reality it would be the result of some analysis of a particular goods market and factor input market. We're simply going to take it as given and remember that these are derived relationships or values and are not necessarily going to be the same for any two situations:

$Y_S = 10 + 3P_Y$ and $Y_D = 42 - P_Y$ These are supply and demand equations (1)

$P_K = 3$ and $P_L = 5$ These are the prices for two factor inputs (2)

$\frac{P_K}{P_L} = 3\frac{L}{K}$ A relationship between the factor inputs (3)

As for as the rules or conditions, we know that the producer is going to try to target a level of production where marginal cost is equal to marginal revenue ($MC = MR$) and that when in a competitive goods market, marginal revenue will simply be equal to the price of the output good ($MR = P_Y$). Notice that I've used Y as the variable denoting the output good; we'll also use this as the variable to indicate the quantity of that good as it's produced – you might more commonly think of that as Q or X, but I want you to get used to thinking of these simply as labels and when we talk about the producer's problem, we often use the variable Y to denote the output good. In this case we're going to use capital (K) and labor (L) as our factor inputs, so we'll simply refer to them as K and L.

We also know that the total revenue of a firm is the price of the output good times the quantity of that good produced ($P_Y * Y = TR$) and we know that the sum of price of each input good times the quantity of the input good is equal to the total cost ($P_K K + P_L L = TC$). Finally, we know that in perfect competition profit is equal to zero such that total revenue minus total cost is equal to zero: we can write this as $TR - TC = \Pi$ and it can be rewritten as $TR - TC = 0$ or $TR = TC$.

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[1] This primer is intended to present an abbreviated discussion of the included economic concepts and is not intended to be a full or complete representation of them or the underlying economic foundations from which they are built.

[2] This primer was developed by Rick Haskell (rick.haskell@utah.edu), Ph.D. Student, Department of Economics, College of Social and Behavioral Sciences, The University of Utah, Salt Lake City, Utah (2013).

We have another rule, or profit maximizing condition, that tells us that the marginal productivity of an input divided by the price of that input is equal to the marginal productivity of the other input divided by the price of the other input. In this case we can write this as:

$$\frac{MP_K}{P_K} = \frac{MP_L}{P_L} \qquad (4)$$

Which can be rewritten as

$$\frac{MP_K}{MP_L} = \frac{P_K}{P_L} \qquad (5)$$

Since this is a profit maximizing condition, it's a target we want to achieve and as such governs much of the rest of what we'll do: remember that it simply tells us something about the marginal productivities of Capital and Labor in respect to their prices. We have another equation (3) that also tells us something about Capital and Labor in respect to their prices so let's rewrite these. Since we know the prices of these inputs we'll first substitute (2) into (3)

$$\frac{3}{5} = 3\frac{L}{K} \qquad (6)$$

Which can be rewritten as

$$K = \left(\frac{5}{3}\right)\left(\frac{3}{1}\right)L = 5L \quad \text{or} \quad K = 5L \qquad (7)$$

We'll think of this as one equation with two unknowns and to solve for the unknowns we need to have another equation with the same two unknowns present.

So let's use the supply and demand equations given (1) and assume we have a market that can be parameterized by them to lead us to that second equation that includes the two unknowns we're looking at

$$Y_S = 10 + 3P_Y \text{ and } Y_D = 42 - P_Y$$

When we set these two equations equal to one another we'll find that $P_Y^* = 8$ and $Y^* = 34$. With this we know that $TR = 8 * 34 = 272$ which also means that $TC = 272$ such that

$$P_K K + P_L L = 272 \qquad (8)$$

If we replace P_K and P_L with the values given in (2) we can rewrite the equation as

$$3K + 5L = 272 \qquad (9)$$

We'll think of this as our production budget. Recall that all of this has simply come about given a set of supply and demand equations and our knowledge of a few simple rules. We can also think of it as one equation with two unknowns, but now we have two of them and from these we're going to find the optimal values of K and L to use in our production process to reach our profit maximizing point of production.

We can substitute (7) into (9) such that

$3(5L) + 5L = 272$ or $15L + 5L = 272$ and $20L = 272$

From here we can rewrite the equation as

$$L = \frac{272}{20} = 13.6 \quad \text{or} \quad L^* = 13.6 \qquad (10)$$

Now we simply need to substitute (10) into (9) to get

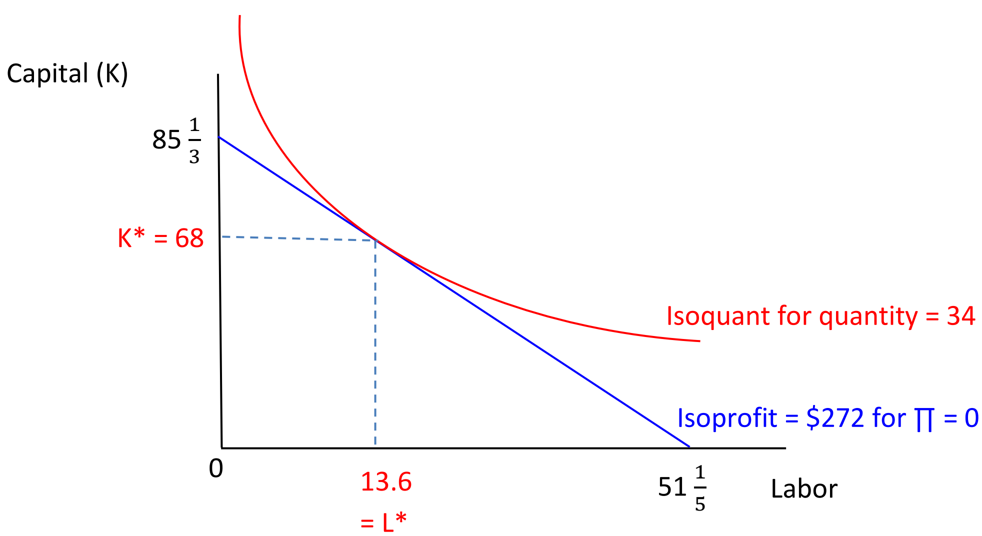
$3K + 5(13.6) = 272$ or with some simple algebra $3K = 204$

And from here we can rewrite the equation as

$$K = \frac{204}{3} = 68 \quad \text{or} \quad K^* = 68$$

So our optimal quantities of K^* and L^* are 68 and 13.6 respectively.

We need to think about how this should look in our graphic model using Isoquant and Isoprofit curves. Remember that on the Isoquant curve we have the same level of output quantity at every point on the curve, but each point represents a different combination of the factor inputs, K and L in this case; so this means each point represents a different technology employed in the production process. Similarly, every point on the Isoprofit curve represents the same level of profit but achieved through different combinations of the factor inputs, again through different technologies.



You might recall that we obtain the intercepts of the respective axes by dividing the budget by the price of the input represented by the axis. If this looks a lot like a consumer optimality model, it should. Think of the Isoprofit as a budget and the Isoquant as the producer's indifference curve, but in this case the producer's utility is being measured at a given quantity of output and we're optimizing it against a budget constraint.